Math 115

Spring 2018

Lecture 27

$? \ a^2 + b^2 = c^2 \ ?$
$?$
$y = mx + b \quad d = rt$

Evaluate $\dfrac{x^2 - 36}{x^2 - 2x - 24}$ for ♥ ♥ ♥

① $x = 0$

$= \dfrac{0^2 - 36}{0^2 - 2(0) - 24}$

$= \dfrac{-36}{-24} = \boxed{\dfrac{3}{2}}$

② $x = -4$

$= \dfrac{(-4)^2 - 36}{(-4)^2 - 2(-4) - 24}$

$= \dfrac{16 - 36}{16 + 8 - 24}$

$= \dfrac{-20}{0}$

$\boxed{\varnothing}, \boxed{\text{undefined}}$

③ $x = 6$

$= \dfrac{6^2 - 36}{6^2 - 2(6) - 24}$
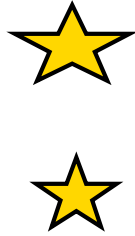
$= \dfrac{36 - 36}{36 - 12 - 24}$

$= \dfrac{0}{0}$

$\boxed{\text{Indeterminate}}$

Simplify

① $\dfrac{x^2 - 4x}{x^2 - 16}$

$= \dfrac{x(x-4)}{x^2 - 4^2}$

$= \dfrac{x(x-4)}{(x+4)(x-4)}$

$= \boxed{\dfrac{x}{x+4}}$

② $\dfrac{x^2 + 8x + 15}{x^2 + 2x - 15}$

$= \dfrac{(x+5)(x+3)}{(x+5)(x-3)}$

$= \boxed{\dfrac{x+3}{x-3}}$
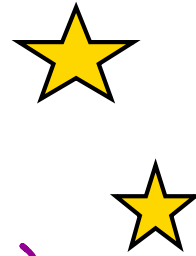
③ $\dfrac{x^2 + 2x - 8}{2x^2 - x - 6}$

$= \dfrac{(x+4)(x-2)}{(2x+3)(x-2)}$

$= \boxed{\dfrac{x+4}{2x+3}}$

④ $\dfrac{xy + 4x + 4y + 16}{xy + 4x + 5y + 20}$

Hint: Factor by grouping

$= \dfrac{x(y+4) + 4(y+4)}{x(y+4) + 5(y+4)}$

$= \dfrac{(y+4)(x+4)}{(y+4)(x+5)}$

$= \boxed{\dfrac{x+4}{x+5}}$

⑤ $\dfrac{x^3+27}{x^3-3x^2+9x}$ Hint: use A^3+B^3 to factor the numerator

$= \dfrac{x^3+3^3}{x(x^2-3x+9)} = \dfrac{(x+3)(x^2-3x+9)}{x(x^2-3x+9)}$

$= \boxed{\dfrac{x+3}{x}}$

⑥ $\dfrac{x^3-4x}{x^3-8}$ Hint: Be aware of GCF, A^2-B^2, A^3-B^3.

$= \dfrac{x(x^2-4)}{x^3-8}$

$= \dfrac{x(x^2-2^2)}{x^3-2^3} = \dfrac{x(x+2)(x-2)}{(x-2)(x^2+2x+4)}$

$= \boxed{\dfrac{x(x+2)}{x^2+2x+4}}$

find all excluded Values: Deno. = 0,
Solve

① $\dfrac{x+1}{x-6}$ $x-6=0$ ② $\dfrac{3x^2}{4x+5}$

$\boxed{x=6}$

$\boxed{E.V.:6}$

$4x+5=0$

$4x=-5$ $\boxed{x=\dfrac{-5}{4}}$

③ $\dfrac{x+10}{x^2-x-6}$

$\boxed{E.V.:\dfrac{-5}{4}}$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$x-3=0$ $x+2=0$ $x=3$ $x=-2$ $\boxed{E.V.:-2,3}$

Simplify:

① $\dfrac{5x-15}{3x+9} \cdot \dfrac{4x+12}{6x-18}$

$= \dfrac{5(x-3)}{3(x+3)} \cdot \dfrac{\overset{2}{4}(x+3)}{\underset{3}{6}(x-3)}$

$= \boxed{\dfrac{10}{9}}$

② $\dfrac{x^2+4x-5}{x^2+7x+10} \div \dfrac{x-1}{x+4}$

$= \dfrac{(x-1)(x+5)}{(x+5)(x+2)} \cdot \dfrac{x+4}{x-1}$

$= \boxed{\dfrac{x+4}{x+2}}$

③ $\dfrac{x^2-x-6}{x^2-2x-8} \cdot \dfrac{x^2+7x+12}{x^2-9}$

$= \dfrac{(x-3)(x+2)}{(x-4)(x+2)} \cdot \dfrac{(x+3)(x+4)}{(x+3)(x-3)}$

$= \boxed{\dfrac{x+4}{x-4}}$

④ $\dfrac{2x^2+7x+3}{x^2-9} \div \dfrac{2x^2+11x+5}{x^2-3x}$

$= \dfrac{2x^2+7x+3}{x^2-9} \cdot \dfrac{x^2-3x}{2x^2+11x+5}$

$= \dfrac{(2x+1)(x+3)}{(x+3)(x-3)} \cdot \dfrac{x(x-3)}{(2x+1)(x+5)}$

$= \boxed{\dfrac{x}{x+5}}$

Adding/Subtracting Rational expressions:

Type I: Same denominator

$\dfrac{4}{9} - \dfrac{1}{9} = \dfrac{4-1}{9} = \dfrac{3}{9} = \boxed{\dfrac{1}{3}}$

$\dfrac{2x+3}{x+3} - \dfrac{x}{x+3} = \dfrac{2x+3-x}{x+3} = \dfrac{x+3}{x+3} = \boxed{1}$

$\dfrac{x^2-3x}{x^2-25} - \dfrac{2x}{x^2-25} = \dfrac{x^2-3x-2x}{x^2-25} = \dfrac{x^2-5x}{x^2-25}$

$= \dfrac{x(x-5)}{(x+5)(x-5)}$

$= \boxed{\dfrac{x}{x+5}}$

Simplify:

① $\dfrac{5m}{m-1} - \dfrac{4m+1}{m-1}$

$= \dfrac{5m-4m-1}{m-1}$

$= \dfrac{m-1}{m-1} = \boxed{1}$

② $\dfrac{x^2-8x}{x-5} + \dfrac{15}{x-5}$

$= \dfrac{x^2-8x+15}{x-5}$

$= \dfrac{(x-5)(x-3)}{x-5}$

$= \boxed{x-3}$

① $\dfrac{x^2}{x-3} - \dfrac{3x}{x-3}$

$= \dfrac{x^2-3x}{x-3}$

$= \dfrac{x(x-3)}{x-3} = \boxed{x}$

② $\dfrac{x^2}{x^2-25} + \dfrac{5x}{x^2-25}$

$= \dfrac{x^2+5x}{x^2-25}$

$= \dfrac{x(x+5)}{(x+5)(x-5)}$

$= \boxed{\dfrac{x}{x-5}}$

Case II: Different denominator

$$\frac{5}{6} - \frac{1}{4} = \frac{5 \cdot 2}{2 \cdot 3 \cdot 2} - \frac{1 \cdot 3}{2 \cdot 2 \cdot 3}$$

$$= \frac{10}{12} - \frac{3}{12} = \frac{10-3}{12} = \boxed{\frac{7}{12}}$$

$$\frac{5}{x^2-9} + \frac{2}{x^2+8x+15}$$

$$= \frac{5(x+5)}{(x-3)(x+3)(x+5)} + \frac{2(x-3)}{(x+5)(x+3)(x-3)}$$

$$= \frac{5(x+5) + 2(x-3)}{(x-3)(x+3)(x+5)} = \frac{\overset{7x+19}{5x+25+2x-6}}{(x-3)(x+3)(x+5)}$$

$$= \boxed{\frac{7x+19}{(x-3)(x+3)(x+5)}}$$

Simplify:

$$\frac{2x}{x^2-16} - \frac{3}{x^2+6x+8}$$

$$= \frac{2x(x+2)}{(x-4)(x+4)(x+2)} - \frac{3(x-4)}{(x+4)(x+2)(x-4)}$$

$$= \frac{2x(x+2) - 3(x-4)}{(x-4)(x+4)(x+2)} = \frac{2x^2+4x-3x+12}{(x-4)(x+4)(x+2)}$$

$$= \boxed{\frac{2x^2+x+12}{(x-4)(x+4)(x+2)}}$$

Simplify: $\dfrac{2}{x^2-36} - \dfrac{1}{x^2+6x}$

$= \dfrac{2 \cdot x}{(x+6)(x-6)\cdot x} - \dfrac{1(x-6)}{x(x+6)(x-6)}$

$= \dfrac{2x}{x(x+6)(x-6)} - \dfrac{x-6}{x(x+6)(x-6)}$

$= \dfrac{2x - x + 6}{x(x+6)(x-6)} = \dfrac{\overset{1}{x+6}}{x(x+6)(x-6)} = \boxed{\dfrac{1}{x(x-6)}}$

$\dfrac{3}{x^2-4} - \dfrac{2}{x^2+5x+6} - \dfrac{1}{x^2+x-6}$

$= \dfrac{3(x+3)}{(x+2)(x-2)(x+3)} - \dfrac{2(x-2)}{(x+2)(x+3)(x-2)} - \dfrac{1(x+2)}{(x+3)(x-2)(x+2)}$

$= \dfrac{3(x+3) - 2(x-2) - 1(x+2)}{(x+2)(x-2)(x+3)} = \dfrac{3x+9-2x+4-x-2}{(x+2)(x-2)(x+3)}$

$= \boxed{\dfrac{11}{(x+2)(x-2)(x+3)}}$